Ex 1 Let $f(x) = e^x$. Find the degree k polynomial, $g_k(x)$, which has $g_k(0) = f(0)$, $g_k'(0) = f'(0)$, $g_k''(0) = f''(0)$,..., $g_k^{(k)}(0) = f^{(k)}(0)$.

Ex 2 Let $f(x) = e^x$. Find the degree k polynomial, $h_k(x)$, which has $h_k(5) = f(5)$, $h_k'(5) = f'(5)$, $h_k''(5) = f''(5)$,..., $h_k^{(k)}(5) = f^{(k)}(5)$.

Q. What if we take k to infinity?

Def: A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$ is called a _____

Q. How do you want to think about power series?

Key Skill:

Def: For the power series $\sum_{n=0}^{\infty} c_n(x-a)^n$...

- The constant "a" is the _____
- The numbers $c_0, c_1, c_2, \ldots$ are the _____

Q. For which values of x does a power series converge?

Ex 3 For which values of x does the power series $\sum_{n=0}^{\infty} x^n$ converge?

Ex 4 For which values of x does the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converge?

Ex 5 For which values of x does the power series $\sum_{n=0}^{\infty} n! \cdot x^n$ converge?

Thm: If I is the set of all x-values for which $\sum_{n=0}^{\infty} c_n(x-a)^n$ converges, there are two possibilities.

(a) The series converges on an interval of "radius" $R \geqslant 0$, in which case I is one of the following: $(a-R, a+R)$, $[a-R, a+R)$, $(a-R, a+R]$, or $[a-R, a+R]$.

(b) The series converges for all x, in which case $I = (-\infty, \infty)$.

Def: In the setting of the above theorem,

- I is called the _____
- R is called the _____

Q. What is R if $I = (-\infty, \infty)$?

Q. Given a power series, how do we find the radius and interval of convergence?

Ex 6 Find the center, radius of convergence, and interval of convergence for $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}5^n}(x-3)^n$.

The center is...

(a) -5
(b) -2
(c) -1
(d) 0
(e) 0.5
(f) 1
(g) 3
(h) 5

Q. How do we find I in general?

(a)

(b)

(c)

Ex 7 Find the interval of convergence for $\sum_{n=0}^{\infty} n^n(x-3)^n$.

Ex 8 Find the interval of convergence for $\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(-3)^k \cdot (2k+1)}$. The center is...

(a) -1 (b) 0 (c) 1 (d) 2

Ex 9 Find the interval of convergence for $\sum_{n=0}^{\infty} \frac{(2x+8)^n}{n!}$. The center is...

(a) -8 (c) -2 (e) 2 (g) 8
(b) -4 (d) 0 (f) 4

Q. How can we tell what a power series converges to?

Q. How can we convert familiar functions to power series?

Ex 10 Find the power series centered at $a = 9$ which converges to $f(x) = \frac{2}{1-x}$ and identify the interval of convergence of the power series.

Ex 11 Find the power series for $f(x) = \frac{x}{4+x^2}$ centered at $a = 0$ and identify the interval of convergence.

Q. What's a helpful form for $f(x)$?

(a) $f(x) = x \cdot \dfrac{1}{4+x^2}$

(b) $f(x) = \dfrac{x}{4} \cdot \dfrac{1}{1+\frac{x^2}{4}}$

(c) $f(x) = \dfrac{x}{-4} \cdot \dfrac{1}{-1-\frac{x^2}{4}}$

(d) $f(x) = \dfrac{x}{4} \cdot \dfrac{1}{1-(-\frac{x^2}{4})}$

Ex 12 Find the power series for $f(x) = \frac{1}{x}$ centered at $a = 1$ and identify the interval of convergence.

Q. What's the most helpful form for $f(x)$?

(a) $f(x) = \dfrac{1}{x}$

(b) $f(x) = \dfrac{1}{1-(1-x)}$

(c) $f(x) = \dfrac{1}{1-(-(x-1))}$

(d) $f(x) = x^{-1}$

Ex 13 Find a power series for $f(x) = \dfrac{9-3x}{x^2-6x}$.

Q. What technique do you think will be helpful here?

(a) Factor the denominator

(b) Partial fraction decomposition

(c) Complete the square in the denominator

(d) Long divide the numerator and the denominator